Doubly heavy baryons $\Omega_{QQ'}$ versus $\Xi_{QQ'}$ in sum rules of nonrelativistic QCD

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(Received 27 June 2000; revised manuscript received 27 November 2000; published 14 May 2001)

In the framework of two-point sum rules of nonrelativistic QCD, the masses and couplings of doubly heavy baryons to the corresponding quark currents are evaluated taking into account Coulomb-like corrections in the system of a doubly heavy diquark as well as the contribution of nonperturbative terms determined by the quark, gluon, mixed condensates, and the product of gluon and quark condensates. The introduction of nonzero light quark mass destroys the factorization of baryon and diquark correlators even at the perturbative level and provides the better convergency of sum rules. We estimate the difference $M_\Omega - M_\Xi = 100\pm 30$ MeV. The ratio of baryonic constants $|Z_\Omega|^2/|Z_\Xi|^2$ is equal to 1.3 ± 0.2 indicating the violation of $SU(3)$ flavor symmetry for the doubly heavy baryons.

DOI: 10.1103/PhysRevD.64.014002 PACS number(s): 12.39.Hg, 14.20.Lq, 14.20.Mr

I. INTRODUCTION

Testing the QCD dynamics of heavy quarks in various conditions provides us with a qualitative and quantitative knowledge that allows us to distinguish fine complex effects caused by the electroweak nature of *CP* violation or physics beyond the standard model. The list of hadrons containing the heavy quarks, as available to the experimental observations and measurements, was recently enriched by a new member, the long-lived B_c meson, in addition to the heavy quarkonia $\bar{b}b$ and $\bar{c}c$ as well as the mesons and baryons with a single heavy quark. This success of the Collider Detector at Fermilab Collaboration in the first observation of the B_c meson [1] was based on the progress of the experimental technique in the reconstruction of rare processes with heavy quarks by the use of vertex detectors. This experience supports the hope to observe other rare long-lived doubly heavy hadrons, i.e., baryons containing two heavy quarks. As expected they have a production rate [2] and lifetime [3] similar to the B_c-meson ones.

In the present paper we investigate the two-point sum rules [4] of nonrelativistic QCD (NRQCD) [5]. The light quark-doubly heavy diquark structure of baryon leads to the definite expressions for baryonic currents written in terms of nonrelativistic heavy quark fields. To relate the nonrelativistic heavy quark correlators to the full QCD ones we need to take into account the hard gluon corrections by means of solving the renormalization-group equation known up to the two-loop accuracy.

The convergency of sum rules results are essentially improved by taking into account a nonzero light quark mass, i.e., in the case we consider in the present paper in contrast with similar studies of two-point sum rules for the doubly heavy baryons with a light massless quark [6,7]. As was mentioned in Ref. [7], a better stability of sum rules can be achieved by destroying the baryon-diquark factorization in the correlators. Indeed, an essentially improved accuracy of estimates was obtained due to taking into account the nonperturbative interactions caused by higher-dimension operators, lifting the diquark-baryon factorization, in contrast to the consideration in Ref. [6], where a significant uncertainty of results was observed in full QCD sum rules with no product of quark and gluon condensates. This uncertainty appears in two quantities. First, the baryon mass, estimated from two scalar correlation functions defining the two-point correlator, has the stability with respect to the variable of sum rules in the single correlator only, while the other correlator results in the value showing no stability [6]. Second, the baryon coupling constants have the stability regions in both correlation functions, however, the estimated optimal values differs by a factor of 2–8 depending on the flavors of quarks composing the baryonic current. This fact shows a bad accuracy in the evaluation of coupling constants, though the dependence on the threshold energy of hadron continuum is quite slow, and it is negligible in comparison with the systematic uncertainty related to the strong deviation of estimates from two correlation functions. So, the stability regions in Ref. [6] were determined from the consideration of coupling constants and used to evaluate the baryon masses, whose estimates are quite stable under the variation of continuum threshold, Ref. [6], while such a method of mass evaluation involves an additional uncertainty. We follow the NRQCD sum rules for the double heavy baryons in Ref. [7], including the higher quark-gluon condensates, which improve the accuracy of estimates, since both correlation functions give close values of baryon coupling constants, and the baryon masses coincide in calculations from two correlators in the region of stability for the coupling constants. We show that for the strange $\Omega_{QQ'}$ baryons the factorization of baryon and diquark correlation functions is broken already in the perturbative limit that allows us to introduce a new criterion for the determination of baryon masses since we observe the stability of sum rules for the masses obtained from both correlators standing in front of two independent Lorentz structures for the spinor field of $\Omega_{QQ'}$.

Moreover our choice of baryonic current is convenient to take into account the α_s/v Coulomb-like corrections [8] inside the doubly heavy diquark.

In Sec. II we describe the scheme of calculation. There we define the currents and represent the spectral densities in the NRQCD sum rules for various operators included into the consideration. The numerical estimates in comparison with the values obtained in potential models are given in Sec. III. The results are summarized in the Conclusion.

II. NONRELATIVISTIC QCD SUM RULES FOR DOUBLY HEAVY BARYONS

A. Description of the method

In order to determine the masses and coupling constants of baryons in sum rules, we consider the two point correlators of interpolating baryon currents. The quantum numbers of doubly heavy diquark in the ground states are given by its spin and parity, so that $j_d^P=1^+$ or $j_d^P=0^+$ (if the identical heavy quarks form the diquark then the scalar state $j_d^P=0^+$ is forbidden). Adding the light quark to form the baryon, we obtain the pair of degenerate states $j^P=1/2^+$ and $j^P=3/2^+$ for the baryons[1] $\Xi_{cc}^{\diamondsuit}$, $\Xi_{bc}^{\diamondsuit}$, $\Xi_{bb}^{\diamondsuit}$ and $\Xi_{cc}^{*\diamondsuit}$, $\Xi_{bc}^{*\diamondsuit}$, $\Xi_{bb}^{*\diamondsuit}$ with the vector diquark, and $j^P=1/2^+$ for the $\Xi_{bc}^{\prime\diamondsuit}$ baryons with the scalar diquark. Unlike the case of baryons with a single heavy quark [9], there is the only independent current component for each ground state. We find

$$J_{\Xi'^{\diamondsuit}_{QQ'}}=[Q^{iT}C\tau\gamma_5Q^{j\prime}]q^k\varepsilon_{ijk},$$

$$J_{\Xi^{\diamondsuit}_{QQ}}=[Q^{iT}C\tau\gamma^mQ^j]\gamma_m\gamma_5q^k\varepsilon_{ijk}, \tag{1}$$

$$J^n_{\Xi^{*\diamondsuit}_{QQ}}=[Q^{iT}C\tau\gamma^nQ^j]q^k\varepsilon_{ijk}+\frac{1}{3}\gamma^n[Q^{iT}C\gamma^mQ^j]\gamma_mq^k\varepsilon_{ijk},$$

where $J^n_{\Xi^{*\diamondsuit}_{QQ}}$ satisfies the spin-3/2 condition $\gamma_nJ^n_{\Xi^{*\diamondsuit}_{QQ}}=0$. The flavor matrix τ is antisymmetric for $\Xi_{bc}^{\prime\diamondsuit}$ and symmetric for $\Xi_{QQ}^{\diamondsuit}$ and $\Xi_{QQ}^{*\diamondsuit}$. Here T means transposition, and C is the charge-conjugation matrix.

To compare the definition of baryonic currents with the full QCD analysis, we represent the expression for the $J_{\Xi'^{\diamondsuit}_{bc}}$ current given in [6]

$$J_{\Xi'^{\diamondsuit}_{bc}}=\{r_1[u^{iT}C\gamma_5c^j]b^k+r_2[u^{iT}Cc^j]\gamma_5b^k+r_3[u^{iT}C\gamma_5\gamma_muc^j]\gamma^\mu b^k\}\varepsilon_{ijk}, \tag{2}$$

so that the NRQCD structure can be obtained by the choice of $r_1=r_2=1$ and $r_3=0$ and the antisymmetric permutation of c and b flavors. This connection can be achieved by the nonrelativistic limit of full QCD spinors of heavy quarks, so that in the leading order of $1/m_Q$ expansion, the ''large'' components of spinors contribute only. Therefore, for the ground states of doubly heavy baryons containing the heavy quarks with the identical flavors, the leading approximation of NRQCD leads to the only structure of baryonic current expressed in terms of nonrelativistic spinors of heavy quark, since the total spin of heavy diquark is fixed by $S=1$ because of the Pauli principle. The corrections of the first $1/m_Q$ order can contribute with the other Lorentz structures, of course. However, we deal with the leading approximation of NRQCD in the present paper. For the ground states of doubly heavy baryons containing the heavy quarks with the different flavors, the 1/2-spin state of baryon can contain the mixture of diquark states with $S=1$ and $S=0$, as it does in full QCD. We perform the separate consideration of these two currents in NRQCD, and such an approach is generally not optimal in full QCD currents. Nevertheless, as we show, in the leading order of NRQCD, there are relations between the masses and coupling constants of baryons because of the spin symmetry, so that the NRQCD does not distinguish these spin states until the spin dependent $1/m_Q$ corrections are taken into account. In addition, as we have already mentioned, the analysis in the sum rules of full QCD was done with a large uncertainty because of a difference in the evaluations of coupling constants from two correlation functions [6], and the authors noted that this uncertainty became large in a region of parameters $r_{1,2}$ defined above, so that this region of bad accuracy is placed in the vicinity of point giving the NRQCD choice in Eq. (2).

The matrix structure of the correlator for two baryonic currents with the spin of 1/2 has the form

$$\Pi(w)=i\int d^4x\, e^{ipx}\langle 0|TJ(x),\bar{J}(0)|0\rangle=\not{v}F_1(w)+F_2(w), \tag{3}$$

where w is defined by $p^2=(\mathcal{M}+w)^2$, $\mathcal{M}=m_Q+m_{Q'}+m_s$, $m_{Q,Q'}$ are the heavy quark masses, and m_s is the strange quark mass. The appropriate definitions of scalar formfactors for the 3/2-spin baryon are given by the following:

$$\Pi_{\mu\nu}(w)=i\int d^4x\, e^{ipx}\langle 0|T\{J_\mu(x),\bar{J}_\nu(0)\}|0\rangle$$
$$=-g_{\mu\nu}[\not{v}\tilde{F}_1(w)+\tilde{F}_2(w)]+\cdots, \tag{4}$$

where we do not concern distinct Lorentz structures. The scalar correlators F can be evaluated in a deep Euclidean region by employing the operator product expansion (OPE) in the framework of NRQCD,

$$F_{1,2}(w)=\sum_d C_d^{(1,2)}(w)O_d, \tag{5}$$

where O_d denotes the local operator with a given dimension d: $O_0=\hat{1}$, $O_3=\langle\bar{q}q\rangle$, $O_4=\langle(\alpha_s/\pi)G^2\rangle,\ldots$, and the functions $C_d(w)$ are the corresponding Wilson coefficients of OPE. For the contribution of quark condensate operator we explore the following OPE up to the terms of the fourth order in x (the derivation is presented in the Appendix):

[1]The superscript $\diamondsuit$ denotes various electric charges depending on the flavor of the light quark.

$$\langle 0|Ts_i^a(x)\bar{s}_j^b(0)|0\rangle = -\frac{1}{12}\delta^{ab}\delta_{ij}\langle\bar{s}s\rangle\left[1+\frac{x^2(m_0^2-2m_s^2)}{16} + \frac{x^4\left[\pi^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle-\frac{3}{2}m_s^2(m_0^2-m_s^2)\right]}{288}\right] + im_s\delta^{ab}x_\mu\gamma_{ij}^\mu\langle\bar{q}q\rangle\left[\frac{1}{48}+\frac{x^2}{24^2}\left(\frac{3m_0^2}{4}-m_s^2\right)\right]. \quad (6)$$

Note that at $m_s\neq 0$ the expansion of the quark condensate gives contributions in both correlators in contrast with the sum rules for $\Xi_{QQ'}$ [7], where putting $m_s=0$ and neglecting the higher condensates, the authors found the factorization of diquark correlator in F_2 and a full baryonic correlator in F_1. This fact was the physical reason for a large uncertainty of the SR method in the consideration of Ref. [7], until the higher-order condensates were included in the calculations.

Indeed, if the light quark is massless, then in NRQCD the light quark condensate contributes to the F_2 correlators, only. This fact has a simple physical explanation: to the leading order the light quark operator can be factorized in the expression for this correlation function of baryonic currents. So, we can write down for the condensate contribution

$$\langle 0|T\{J(x),\bar{J}(0)\}|0\rangle = \langle 0|T\{q_i^a(x)\bar{q}_i^{\,a}(0)\}|0\rangle \times\frac{\hat{1}}{12}\langle 0|T\{J_d^j(x),\bar{J}_d^j(0)\}|0\rangle+\cdots,$$

where $J_d^j(x)$ denotes the appropriate diquark current with the color index j, as it is defined by the structure of chosen baryonic currents. Then, we see that the restriction by the first term independent of x in the expansion for the quark correlator results in the independent contribution of the diquark correlator to the baryonic one. Since the diquark correlator is isolated in F_2, the corresponding NRQCD sum rules lead to the evaluation of diquark mass and couplings from F_2, which are different from the estimation of baryon masses and couplings from F_1. The positive point is the possibility to calculate the binding energy for the doubly heavy baryons $\bar{\Lambda}=M_\Xi-\mathcal{M}_{\rm diq}$ (see Ref. [7]). The disadvantage is the large uncertainty of NRQCD sum rules at this stage, since the various correlation functions lead to the different results. In the sum rules of full QCD, various choices of parameters in the definitions of baryonic currents result in an admixture of pure diquark correlator in various functions, so that the accuracy of estimations is low. Say, the characteristic ambiguity in the evaluation of baryon mass in full QCD is about 300 MeV, i.e., the value close to the expected estimate of $\bar{\Lambda}$. The analysis in the framework of NRQCD makes this result to be not unexpectable. Moreover, it is quite evident that the introduction of interactions between the light quark and the doubly heavy diquark destroys the factorization of the diquark correlator. Indeed, we see that due to both the higher terms in the expansion for the light quark condensate and the nonzero quark mass, the diquark factorization is explicitly broken, which has to result in a better accuracy of estimates obtained from F_1 and F_2. Below we show numerically that this fact is valid.

We write down the Wilson coefficient in front of unity and quark-gluon operators by making use of the dispersion relation over w,

$$C_d(w)=\frac{1}{\pi}\int_0^\infty\frac{\rho_d(\omega)d\omega}{\omega-w}, \quad (7)$$

where ρ denotes the imaginary part in the physical region of NRQCD.

To relate the NRQCD correlators to the real hadrons, we use the dispersion representation for the two point function with the physical spectral density given by the appropriate resonance and continuum part. The coupling constants of baryons are defined by the following expressions:

$$\langle 0|J(x)|\Xi(\Omega)^\diamond_{QQ}(p)\rangle = iZ_{\Xi(\Omega)^\diamond_{QQ}}u(v,M_{\Xi(\Omega)})e^{ipx},$$

$$\langle 0|J^m(x)|\Xi(\Omega)^{*\diamond}_{QQ}(p,\lambda)\rangle = iZ_{\Xi(\Omega)^{*\diamond}_{QQ}}u^m(v,M_{\Xi(\Omega)})e^{ipx},$$

where the spinor field with the four-velocity v and mass M (the mass of baryon) satisfies the equation $\not{v}u(v,M)=u(v,M)$, and $u^m(v,M)$ denotes the transversal spinor.

Then we use the nonrelativistic expressions for the physical spectral functions

$$\rho_{1,2}^{\rm phys}(\omega)=\frac{M}{2\mathcal{M}}|Z|^2\delta(\bar{\Lambda}-\omega), \quad (8)$$

where we have performed the substitution $\delta(p^2-M^2)\to(1/2\mathcal{M})\delta(\bar{\Lambda}-\omega)$; here $\bar{\Lambda}$ is the binding energy of baryon and $M=\mathcal{M}+\bar{\Lambda}$. The nonrelativistic dispersion relation for the hadronic part of sum rules has the form

$$\int\frac{\rho_{1,2}^{\rm phys}d\omega}{\omega-w}=\frac{1}{2\mathcal{M}}\frac{|Z|^2}{\bar{\Lambda}-w}. \quad (9)$$

We suppose that the continuum densities starting from the threshold $\omega_{\rm cont}$ are modeled by the NRQCD expressions. Then, in the sum rules equalizing the correlators in NRQCD and those given by the physical states, we assume the model of continuum given by the calculated perturbative term. This model cannot be exact because of binding effects as well as the truncation of perturbative expansion in the given order of α_s. Therefore, the integration above $\omega_{\rm cont}$ cannot be strictly canceled, and the model introduces the implicit dependence on the choice of value $\omega_{\rm cont}$. This dependence causes an uncertainty, which is not essential in comparison with uncertainties following from other sources and the variation of quark masses. Further, we write down the correlators in the deep underthreshold point of $w=-\mathcal{M}+t$ with $t\to 0$.

Now the sum rules in the scheme of moments, with respect to t, can be written down as follows:

$$\frac{1}{\pi}\int_0^{\omega_{\rm cont}}\frac{\rho_{1,2\,d\omega}}{(\omega+\mathcal{M})^n}=\frac{M}{2\mathcal{M}}\,\frac{|Z|^2}{M^n}, \tag{10}$$

where ρ_j contains the contributions given by various operators in OPE for the corresponding scalar functions F_j. Introducing the following notation for the nth moment of two point correlation function:

$$\mathcal{M}_n=\frac{1}{\pi}\int_0^{\omega_{\rm cont}}\frac{\rho(\omega)d\omega}{(\omega+\mathcal{M})^{n+1}}, \tag{11}$$

we can obtain the estimates of baryon mass M, for example, as the following:

$$M[n]=\frac{\mathcal{M}_n}{\mathcal{M}_{n+1}}, \tag{12}$$

and the coupling is determined by the expression

$$|Z[n]|^2=\frac{2\mathcal{M}}{M}\mathcal{M}_n M^{n+1}, \tag{13}$$

where we see the dependence of sum rule results on the scheme parameter.

B. Calculating the spectral densities

In this section we present analytical expressions for the perturbative spectral functions in the NRQCD approximation. The evaluation of spectral densities involves the standard use of Cutkosky rules [10], with some modification motivated by NRQCD:

$$\text{heavy quark:}\quad \frac{1}{p_0-(m+\mathbf{p}^2/2m)}\to 2\pi i\,\delta[p_0-(m+\mathbf{p}^2/2m)],$$

$$\text{light quark:}\quad \frac{1}{p^2-m^2}\to 2\pi i\,\delta(p^2-m^2).$$

We derive the spin-symmetry relations for all the spectral densities, due to the fact that in the leading order of the heavy quark effective theory, the spins of heavy quarks are decoupled, so

$$\rho_{1,\Omega_{QQ}^{\diamond}}=3\rho_{1,\Omega'^{\diamond}_{QQ'}}=3\rho_{1,\Omega^{*\diamond}_{QQ}}, \tag{14}$$

$$\rho_{2,\Omega_{QQ}^{\diamond}}=3\rho_{2,\Omega'^{\diamond}_{QQ'}}=3\rho_{2,\Omega^{*\diamond}_{QQ}}, \tag{15}$$

and we have the following relation for the baryon couplings in NRQCD:

$$|Z_\Omega|^2=3|Z_{\Omega'}|^2=3|Z_{\Omega*}|^2. \tag{16}$$

Using the smallness of the strange quark mass we use the following expansions in m_s for the perturbative spectral densities standing in front of unity operator [$m_{QQ'}=m_Q m_{Q'}/(m_Q+m_{Q'})$ is the reduced diquark mass, $\mathcal{M}_{\rm diq}=m_Q+m_{Q'}$]:

$$\rho_{1,\Omega'^{\diamond}_{QQ'}}(\omega)=\frac{\sqrt{2}(m_{QQ'}\omega)^{3/2}}{15\,015\pi^3(\mathcal{M}_{\rm diq}+\omega)^3}[\eta_{1,0}(\omega)+m_s\eta_{1,1}(\omega)+m_s^2\eta_{1,2}(\omega)], \tag{17}$$

where we have found

$$\eta_{1,0}(\omega)=16\omega^2(429\mathcal{M}_{\rm diq}^3+715\mathcal{M}_{\rm diq}^2\omega+403\mathcal{M}_{\rm diq}\omega^2+77\omega^3),$$

$$\eta_{1,1}(\omega)=104\omega(231\mathcal{M}_{\rm diq}^3+297\mathcal{M}_{\rm diq}^2+121\mathcal{M}_{\rm diq}\omega^2+15\omega^3), \tag{18}$$

$$\eta_{1,2}(\omega)=\frac{10}{(\mathcal{M}_{\rm diq}+\omega)^2}(3003\mathcal{M}_{\rm diq}^5+9009\mathcal{M}_{\rm diq}^4\omega+9438\mathcal{M}_{\rm diq}^3\omega^2+4290\mathcal{M}_{\rm diq}^2\omega^3+871\mathcal{M}_{\rm diq}\omega^4+77\omega^5).$$

The first term of this expansion reproduces the result obtained in Ref. [7]. A new feature is the appearance of nonzero perturbative $\rho_{2,\Omega'^{\diamond}_{QQ'}}$ density, which is proportional to m_s

$$\rho_{2,\Omega'^{\diamond}_{QQ'}}(\omega)=\frac{2\sqrt{2}\omega(m_{QQ'}\omega)^{3/2}m_s}{105\pi^3(\mathcal{M}_{\rm diq}+\omega)^2}(\eta_{2,0}+m_s\eta_{2,1}+m_s^2\eta_{2,2}, \tag{19}$$

and

$$\eta_{2,0}=42\omega(\mathcal{M}_{\rm diq}^2+48\mathcal{M}_{\rm diq}\omega+14\omega^2),$$

$$\eta_{2,1}=3(35\mathcal{M}_{\rm diq}^2+28\mathcal{M}_{\rm diq}\omega+5\omega^2), \tag{20}$$

$$\eta_{2,2}=\frac{1}{(\mathcal{M}_{\rm diq}+\omega)^2}(105\mathcal{M}_{\rm diq}^3+315\mathcal{M}_{\rm diq}^2\omega+279\mathcal{M}_{\rm diq}\omega^2+77\omega^3).$$

The account for the Coulomb-like interaction leads to the finite renormalization of the diquark spectral densities before the integration over the diquark invariant mass by the introduction of Sommerfeld factor $\mathbf{C}$, so that

$$\rho^C_{\rm diquark}=\rho^{\rm bare}_{\rm diquark}\mathbf{C} \tag{21}$$

with

$$\mathbf{C}=\frac{2\pi\alpha_s}{3v_{QQ'}}\left[1-\exp\left(-\frac{2\pi\alpha_s}{3v_{QQ'}}\right)\right]^{-1}, \tag{22}$$

where v_{12} denotes the relative velocity of heavy quarks inside the diquark, and we have taken into account the color antitriplet structure of diquark. The relative velocity is given by the following expression:

$$v_{QQ'}=\sqrt{1-\frac{4m_Q m_{Q'}}{Q^2-(m_Q-m_{Q'})^2}}, \tag{23}$$

where Q^2 is the square of heavy diquark four momentum. In NRQCD we take the limit of low velocities, so that denoting the diquark invariant mass squared by $Q^2=(\mathcal{M}_{\rm diq}+\epsilon)^2$, we find

$$\mathbf{C}=\frac{2\pi\alpha_s}{3v_{QQ'}},\quad v_{QQ'}^2=\frac{\epsilon}{2m_{QQ'}},$$

at $\epsilon\ll m_{QQ'}$.

The corrected spectral densities are equal to

$$\rho_1^{\mathbf{C}}(\omega)=\frac{m_{QQ'}^2\alpha_s\omega(2\mathcal{M}_{\rm diq}+\omega)}{6\pi^2(\mathcal{M}_{\rm diq}+\omega)^3}(\eta_{1,0}^{\mathbf{C}}+m_s\eta_{1,1}^{\mathbf{C}}+m_s^2\eta_{1,2}^{\mathbf{C}}), \tag{24}$$

where

$$\eta_{1,0}^{\mathbf{C}}=(2\mathcal{M}_{\rm diq}+\omega)^2\omega^2,$$

$$\eta_{1,1}^{\mathbf{C}}=\frac{3(2\mathcal{M}_{\rm diq}+\omega)\omega}{(\mathcal{M}_{\rm diq}+\omega)}(4\mathcal{M}_{\rm diq}^3+6\mathcal{M}_{\rm diq}^2\omega+4\mathcal{M}_{\rm diq}\omega^2+\omega^3), \tag{25}$$

$$\eta_{1,2}^{\mathbf{C}}=\frac{1}{(\mathcal{M}_{\rm diq}+\omega)^2}(12\mathcal{M}_{\rm diq}^4+24\mathcal{M}_{\rm diq}^3\omega+32\mathcal{M}_{\rm diq}^2\omega^2+20\mathcal{M}_{\rm diq}\omega^3+5\omega^4).$$

We see that the first term again reproduces the result of Ref. [7]. For the $\rho^{\mathbf{C}}_{2,\Omega_{QQ'}^{\diamond}}$ we find

$$\rho_2^{\mathbf{C}}=\frac{m_s m_{QQ'}^2(2\mathcal{M}_{\rm diq}+\omega)\omega\alpha_s}{2\pi(\mathcal{M}_{\rm diq}+\omega)^2}(\eta_{2,0}^{\mathbf{C}}+m_s\eta_{2,1}^{\mathbf{C}}+m_s^2\eta_{2,2}^{\mathbf{C}}), \tag{26}$$

$$\eta_{2,0}^{\mathbf{C}}=(2\mathcal{M}_{\rm diq}+\omega)\omega,$$

$$\eta_{2,1}^{\mathbf{C}}=\frac{2}{\mathcal{M}_{\rm diq}+\omega}(2\mathcal{M}_{\rm diq}^2+2\mathcal{M}_{\rm diq}\omega+\omega^2), \tag{27}$$

$$\eta_{2,0}^{\mathbf{C}}=\frac{2}{(\mathcal{M}_{\rm diq}+\omega)^2}(2\mathcal{M}_{\rm diq}^2+2\mathcal{M}_{\rm diq}\omega+\omega^2).$$

The use of these expansions numerically leads to very small deviations from the exact integral representations of spectral densities (about 0.5%), but they are more convenient in calculations.

The contribution to the moments of the spectral densities determined by the light quark condensate can be calculated by the exploration of Eq. (6),

$$\mathcal{M}_{qq}^{(1)}(n)=-\frac{(n+1)!}{n!}\mathcal{P}_1\mathcal{M}^{\rm diq}(n+1)+\frac{(n+3)!}{n!}\mathcal{P}_3\mathcal{M}^{\rm diq}(n+3),$$
$$\mathcal{M}_{qq}^{(2)}=\mathcal{P}_0\mathcal{M}^{\rm diq}(n)-\frac{(n+2)!}{n!}\mathcal{P}_2\mathcal{M}^{\rm diq}(n+2)+\frac{(n+4)!}{n!}\mathcal{P}_4\mathcal{M}^{\rm diq}(n+4), \tag{28}$$

where we have introduced the coefficients of expansion in x by $\mathcal{P}_i$ [see Eq. (6) and Appendix]. The nth moment of two-point correlator function of diquark is denoted by $\mathcal{M}^{\rm diq}(n)$. Then the diquark spectral density takes the following form:

$$\rho_{\rm diq}=\frac{12\sqrt{2}m_{QQ'}^{3/2}\sqrt{\omega}}{\pi}, \tag{29}$$

which has to be multiplied by the Sommerfeld factor $\mathbf{C}$, wherein the variable ϵ is substituted by ω, since in this case there is no integration over the quark-diquark invariant mass. This corrected density is

$$\rho_{\rm diq}^{\mathbf{C}}=\frac{48\pi\alpha_s m_{QQ'}^2}{3}, \tag{30}$$

and it is independent of ω.

The corrections due to the gluon condensate are given by the density

$$\rho_1^{G^2}(\omega)=\frac{(m_Q^2+m_{Q'}^2+11m_Qm_{Q'})m_{QQ'}^{5/2}\sqrt{\omega}}{21\times2^{10}\sqrt{2}\pi m_Q^2m_{Q'}^2(\mathcal{M}_{\rm diq}+\omega)^2}\times(\eta_{1,0}^{G^2}+m_s\eta_{1,1}^{G^2}+m_s^2\eta_{1,2}^{G^2}), \tag{31}$$

with

$$\eta_{1,0}^{G^2}=84\mathcal{M}_{\rm diq}^3+196\mathcal{M}_{\rm diq}^2\omega+133\mathcal{M}_{\rm diq}\omega^2+11\omega^3,$$
$$\eta_{1,1}^{G^2}=-\frac{2(210\mathcal{M}_{\rm diq}^3+70\mathcal{M}_{\rm diq}^2\omega+21\mathcal{M}_{\rm diq}\omega^2+3\omega^3)}{\mathcal{M}_{\rm diq}+\omega}, \tag{32}$$
$$\eta_{1,2}^{G^2}=-\frac{2(210\mathcal{M}_{\rm diq}^3+70\mathcal{M}_{\rm diq}^2\omega+21\mathcal{M}_{\rm diq}\omega^2+3\omega^3)}{(\mathcal{M}_{\rm diq}+\omega)^2},$$

where we again make the expansion in m_s. In the case of nonzero quark mass we get the nonzero density proportional to m_s

$$\rho_2^{G^2}(\omega)=\frac{m_s(m_Q^2+m_{Q'}^2+11m_Qm_{Q'})m_{QQ'}^{5/2}\sqrt{\omega}}{3\times2^9\sqrt{2}\pi m_Q^2m_{Q'}^2(\mathcal{M}_{\rm diq}+\omega)}\times(\eta_{2,0}^{G^2}+m_s\eta_{2,1}^{G^2}), \tag{33}$$

with

$$\eta_{2,0}^{G^2} = -(9\mathcal{M}_{\rm diq}+\omega),$$
$$\eta_{2,1}^{G^2} = \frac{9\mathcal{M}_{\rm diq}+\omega}{\mathcal{M}_{\rm diq}+\omega}. \quad (34)$$

For the product of condensates $\langle \bar{q}q\rangle\langle(\alpha_s/\pi)G^2\rangle$, wherein the gluon fields are connected to the heavy quarks, it is convenient to compute the contribution to the two-point correlation function itself. We have found

$$F_2^{\bar{q}qG^2}(\omega) = -\frac{m_{QQ'}^{5/2}(m_Q^2+m_{Q'}^2+11m_Q m_{Q'})}{2^9\sqrt{2}\pi m_Q m_{Q'}(-\omega)^{5/2}}, \quad (35)$$

and we put $F_1^{\bar{q}qG^2}(\omega)=0$, since we have restricted the dimension of condensate operators, while the corresponding term in F_1 would appear in the fifth order in expansion (6). The result is given in the form, which allows the analytical continuation over $\omega=-\mathcal{M}+w$.

C. Matching with full QCD

To connect the NRQCD sum rules to the baryonic couplings in full QCD we have to use the relation

$$J^{\rm QCD} = \mathcal{K}_J(\alpha_s,\mu_{\rm soft},\mu_{\rm hard})J^{\rm NRQCD},$$

where the coefficient $\mathcal{K}_J(\alpha_s,\mu_{\rm soft},\mu_{\rm hard})$ depends on the soft normalization scale $\mu_{\rm soft}$. The $\mathcal{K}$ factor obeys the matching condition at the hard scale $\mu_{\rm hard}=\mathcal{M}_{\rm diq}$ and is determined by the anomalous dimensions of effective baryonic currents, which are independent of the diquark spin in the leading order. They are known up to the two-loop accuracy [11]. In our consideration, we use the one-loop accuracy, since the subleading corrections in the first α_s order are not available yet. So

$$\gamma = \frac{d\ln\mathcal{K}_J(\alpha_s,\mu_{\rm soft},\mu_{\rm hard})}{d\ln(\mu)} = \sum_{m=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^m \gamma^{(m)},$$
$$\gamma^{(1)} = [-2C_B(3a-3)+3C_F(a-2)], \quad (36)$$

where $C_F=(N_c^2-1)/2N_c$, $C_B=(N_c+1)/2N_c$, for $N_c=3$, and a is the gauge parameter. In the Feynman gauge $a=1$, and we get $\gamma^{(1)}=-4$. So, in the leading logarithmic approximation and to the one-loop accuracy we find

$$\mathcal{K}_J(\alpha_s,\mu_{\rm soft},\mu_{\rm hard}) = \left(\frac{\alpha_s(\mu_{\rm hard})}{\alpha_s(\mu_{\rm soft})}\right)^{\gamma^{(1)}/2\beta_0}, \quad (37)$$

where $\beta_0=11-2/3N_F=9$. Further, we determine the soft normalization scale for the NRQCD estimates by the average momentum transfer inside the doubly heavy diquark, so that $\mu_{\rm soft}^2=2m_{QQ'}T_{\rm diq}$, where $T_{\rm diq}$ is the kinetic energy in the system of two heavy quarks, which is phenomenologically independent of the heavy quark flavors and approximately equal to 0.2 GeV [12]. Then, the coefficients $\mathcal{K}_J$ are equal to

$$\mathcal{K}_{\Omega_{cc}}\approx 1.95, \quad \mathcal{K}_{\Omega_{bc}}\approx 1.52, \quad \mathcal{K}_{\Omega_{bb}}\approx 1.30, \quad (38)$$

with the characteristic uncertainty of about 10% basically due to the variation of hard and soft scale points $\mu_{\rm hard,\,soft}$. Note that the values of $\mathcal{K}_J$ do not change the estimates of baryon masses, but they are essential in the evaluation of baryon couplings [13].

III. NUMERICAL RESULTS

Evaluating the two-point sum rules, we explore the scheme of moments. We point out the well-known fact that an essential part of uncertainties is caused by the variation of heavy quark masses. Indeed, the results of sum rules for the systems containing two heavy quarks strongly depend on the choice of masses, and this fact allows one to pin down the values of masses with a high precision [14]. In order to avoid effects caused by a slow convergency of perturbative expansion in QCD or a renormalon ambiguity [15], the authors of Ref. [14] performed the analysis in the schemes with the short-distance running heavy quark masses or the quantities, which are defined in the way free of renormalon contributions. These infrared stable masses have a slow variation with respect to the normalization point and the order of α_s in contrast to the perturbative pole mass of heavy quark. So, these quantaties determine the thereshold of quark loop contribution independently of the order of perturbative calculations in α_s.

To the given order in α_s for the NRQCD sum rules, we use the leading quark loop approximation to account for the Coulomb exchange between the heavy quarks. At this stage the heavy quark masses and Coulomb coupling constants are strictly fixed by the data on the charmonium and bottomonium leptonic constants and the masses as described by the QCD sum rules to the same accuracy. The stability or convergency of the sum-rule method applied to these heavy quarkonia,[2] results in the masses of quarks, which agree well with the values of heavy quark masses defined as free of infrared contributions; the potential subtracted mass

$$m_b^{PS} = 4.60\pm 0.11\ \text{GeV},$$

and the kinetic mass

$$m_b^{\rm kin} = 4.56\pm 0.06\ \text{GeV},$$

both obtained in strict three-loop analyses of QCD sum rules for the bottomonium [14]. The corresponding *1S* mass defined by A. Hoang has a slightly larger value. These mass values are dependent of the normalization point, which was chosen in the range of 1–2 GeV. In the quark loop calculations for the bottomonium (see Fig. 1) and charmonium we find the optimal values

$$m_c = 1.40\pm 0.03\ \text{GeV}, \quad m_b = 4.60\pm 0.02\ \text{GeV},$$

[2]We have required that the ratio of initial moments for the spectral densities calculated over the data and in QCD sum rules would be stable.

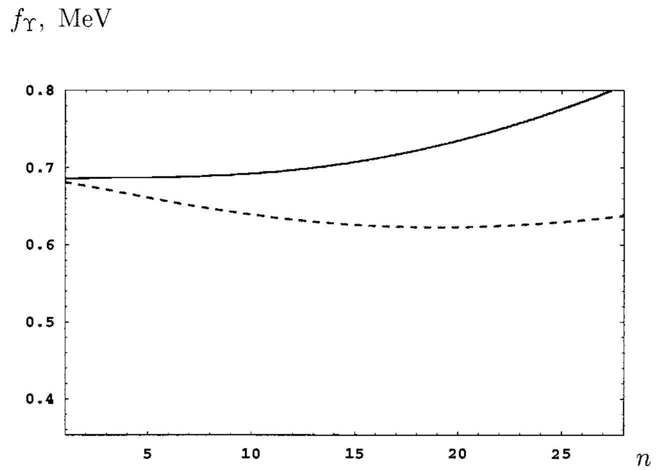

FIG. 1. The leptonic constant of Υ in two-point sum rules in the scheme of moments of spectral density; the dashed line gives the result at $m_b = 4.63$ GeV, while the solid line does at $m_b = 4.59$ GeV.

where possible systematic errors are not included. We see a good agreement between the estimates of bottom quark mass in Ref. [14] and the values following from the accepted approximation. The analysis for the charmed quark mass (see the reference to Narison in Ref. [14]) is not so accurate, though the agreement is also good.

Nevertheless, we slightly enlarge the region of mass variation, so that we accept

$$m_c = 1.35 - 1.45\ \text{GeV}, \quad m_b = 4.56 - 4.64\ \text{GeV}.$$

The same sum rules are also explored to estimate the couplings determining the Coulomb-like interactions inside the heavy quarkonia

$$\alpha_s(b\bar{b}) = 0.37, \quad \alpha_s(c\bar{b}) = 0.45, \quad \alpha_s(c\bar{c}) = 0.60, \quad (39)$$

since they fix the absolute normalization of corresponding leptonic constants known experimentally. The calculated values of coupling constant characteristics for the charmonium and bottomonium are consistent with the estimates given in Ref. [16].

Since the squared size of the diquark is two times larger than that of the meson, the effective Coulumb constants have to be rescaled according to the equation of evolution in QCD. We use the one-loop evolution equation

$$\alpha_s(QQ') = \frac{\alpha_s(Q\bar{Q}')}{1 - (9/4\pi)\alpha_s(Q\bar{Q}')\ln 2}.$$

So,

$$\alpha_s(bb) = 0.45, \quad \alpha_s(bc) = 0.58, \quad \alpha_s(cc) = 0.85. \quad (40)$$

The values of condensates are taken in the ranges $\langle \bar{q}q \rangle = -(0.26\text{–}0.27\ \text{GeV})^3$, $m_0^2 = 0.75\text{–}0.85\ \text{GeV}^2$, $\langle (\alpha_s/\pi) G^2 \rangle = (1.7\text{–}1.8) \times 10^{-2}\ \text{GeV}^4$.

The modern review on the values of light quark masses and condensates can be found in Ref. [17], where we read off $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.7 \pm 0.2$, $(m_u + m_d)[1\ \text{GeV}] = 13.1 \pm 1.5 \pm 1.3$ MeV, and $m_s(1\ \text{GeV}) = 166.7 \pm 18.8$ MeV. We use the variation of parameters in the range, which is in a good agreement with values given above as well as with the conservative estimates: $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8 \pm 0.2$ that corresponds to the variations of the sum $(m_u + m_d)[1\ \text{GeV}] = 12\text{–}14$ MeV [18]. The strange quark mass is taken in the range $m_s = 150\text{–}200$ MeV.

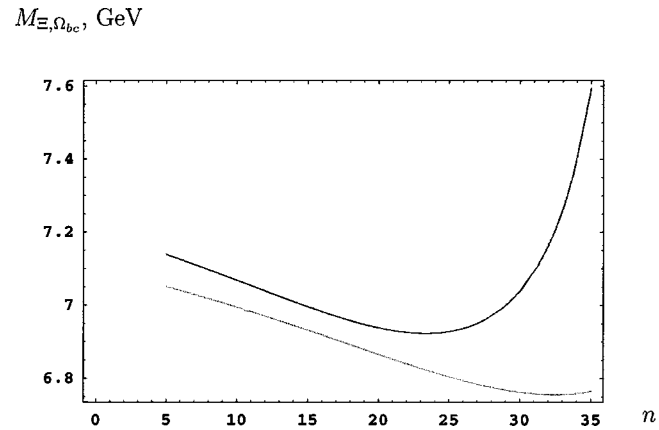

FIG. 2. The Ξ_{bc} (lower curve) and Ω_{bc} (upper curve) masses obtained in the NRQCD sum rules from the first correlator F_1.

So, we have described the set of parameters entering the scheme of calculations. In Figs. 2–4 we present the results of the two-point sum rules for the masses of Ξ_{bc} and Ω_{bc} (the figures for the other baryons are similar). For the Ω_{bc} baryons one can observe the stability of mass with respect to the changing of the moment numbers in both correlators. We suppose it is connected with the destroying of diquark-Ω baryon factorization in the perturbative limit in contrast to the Ξ baryons. The stability regions for F_1 and F_2 do not coincide because the contributions of higher-dimension operators become valuable at the different numbers of moments. However, the quantity $1/2(M_1[n] + M_2[n])$ has the larger stability region, and we explore this fact to determine the Ω baryons masses as well as that of Ξ baryons. The theoretical uncertainties in the Ω-baryon masses are mainly determined by the difference between the values of baryon masses at the regions of stability.

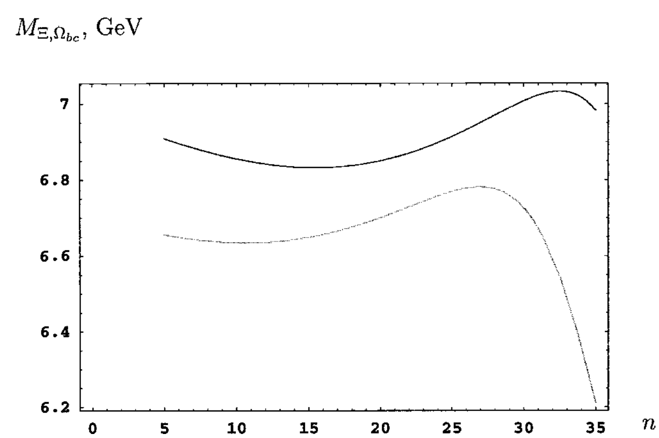

FIG. 3. The Ξ_{bc} (lower curve) and Ω_{bc} (upper curve) masses obtained in the NRQCD two-point sum rules from the second correlator F_2.

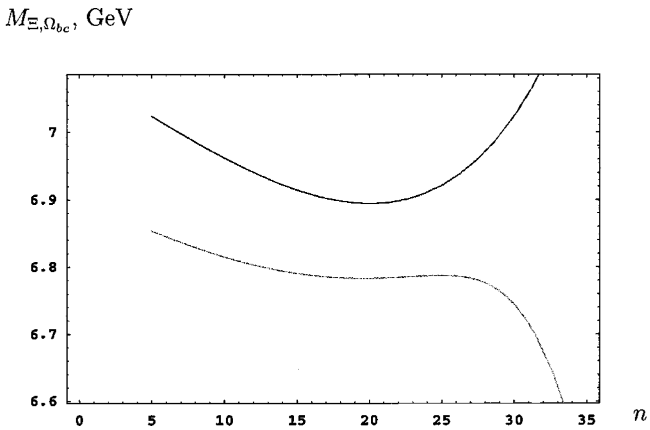

FIG. 4. The Ξ_{bc} (lower curve) and Ω_{bc} (upper curve) masses obtained by averaging the results from both correlators.

We have shown the variation of mass estimates for the Ω_{bb} baryon from the correlation functions $F_{1,2}$ versus the mixed condensate value m_0^2 in Fig. 5 and versus the strange quark mass in Fig. 6.

Then, we investigate the difference between the masses $1/2[(M_{1,\Omega}+M_{2,\Omega})-(M_{1,\Xi}+M_{2,\Xi})]$ shown in Fig. 7. In our scheme of baryon masses determination, this quantity has the meaning of the difference between the Ω and Ξ baryon masses. It has the large region of stability and is determined with a good precision. We obtain

$$\begin{aligned}\Delta M &= M_{\Omega_{bb}} - M_{\Xi_{bb}}\\ &= M_{\Omega_{cc}} - M_{\Xi_{cc}}\\ &= M_{\Omega_{bc}} - M_{\Xi_{bc}}\\ &= 100 \pm 30\,\text{MeV}.\end{aligned}$$

The uncertainty in the Ξ-baryons masses are determined through the uncertainty in the Ω-baryons masses and that of

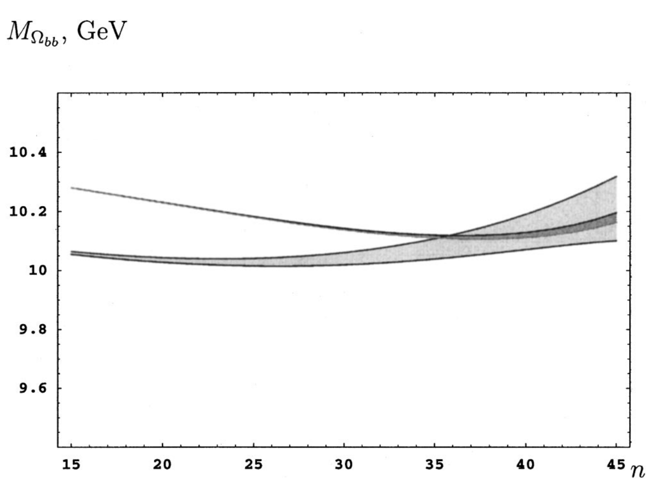

FIG. 5. The mass of Ω_{bb} baryon in two-point sum rules in the scheme of moments of spectral density from two correlation functions F_1 and F_2 (upper and lower shaded regions) with the variation of $m_0^2 = 0.75$–$0.80\,\text{GeV}^2$.

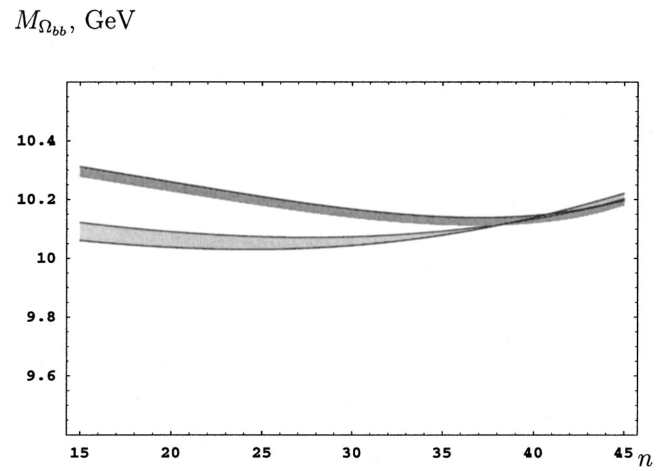

FIG. 6. The mass of Ω_{bb} baryon in two-point sum rules in the scheme of moments of spectral density from two correlation functions F_1 and F_2 (upper and lower shaded regions) with the variation of $m_s = 0.15$–$0.20\,\text{GeV}$.

ΔM. So, for the masses we find the following results:

$$\begin{aligned}&M_{\Omega_{cc}} = 3.65 \pm 0.07\,\text{GeV}, \quad M_{\Xi_{cc}} = 3.55 \pm 0.08\,\text{GeV},\\ &M_{\Omega_{bc}} = 6.89 \pm 0.07\,\text{GeV}, \quad M_{\Xi_{bc}} = 6.79 \pm 0.08\,\text{GeV},\\ &M_{\Omega_{bb}} = 10.09 \pm 0.07\,\text{GeV}, \quad M_{\Xi_{bb}} = 10.00 \pm 0.08\,\text{GeV}.\end{aligned} \tag{41}$$

The obtained values are in agreement with the calculations in the framework of nonrelativistic potential models [19,20], though the models based on the calculation of three-body systems with the pair interactions [20] give slightly higher values of masses. In Ref. [7] the other method of baryon mass determination was used, since the quantities $M_{1,\Xi}$ and $M_{2,\Xi}$, separately, have no good stability in the sum rules. So, the difference of $M_1 - M_2$ close to zero was stable. The use of $1/2(M_1+M_2)$ stability criterion results in the $\Xi_{QQ'}$ masses coinciding with those of Ref. [7] up to 10 MeV. Figures 8 and 9 show the dependence of baryon couplings calculated in the moment scheme of NRQCD sum rules. Numerically, we find

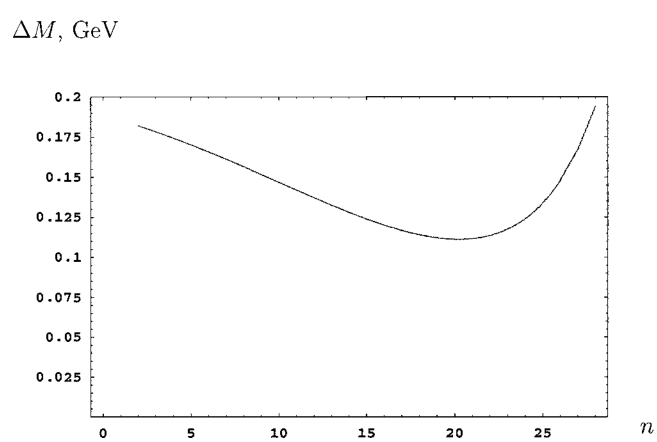

FIG. 7. The mass difference $\Delta M = M_{\Omega_{bc}} - M_{\Xi_{bc}}$ obtained from the results shown in Fig. 4.

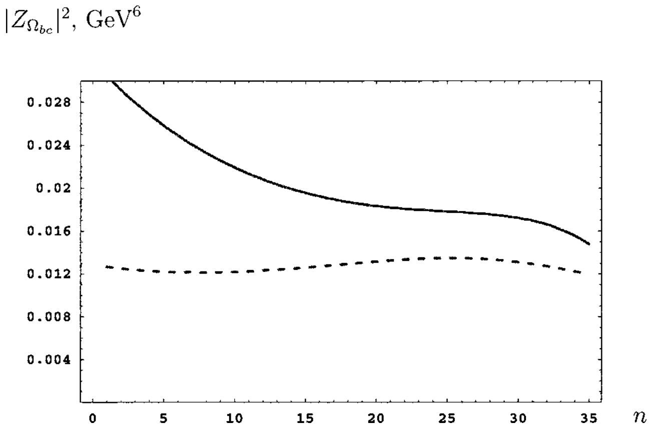

FIG. 8. The coupling $|Z^{(1,2)}_{\Omega_{bc}}|^2$ calculated in NRQCD sum rules for the form factors F_1 and F_2 (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.

$$
\begin{aligned}
|Z_{\Omega_{cc}}|^2 &= (10.0 \pm 1.4) \times 10^{-3}\ \mathrm{GeV}^6,\\
|Z_{\Xi_{cc}}|^2 &= (7.2 \pm 1.0) \times 10^{-3}\ \mathrm{GeV}^6,\\
|Z_{\Omega_{bc}}|^2 &= (15.6 \pm 1.9) \times 10^{-3}\ \mathrm{GeV}^6,\\
|Z_{\Xi_{bc}}|^2 &= (11.6 \pm 1.2) \times 10^{-3}\ \mathrm{GeV}^6,\\
|Z_{\Omega_{bb}}|^2 &= (6.0 \pm 1.0) \times 10^{-2}\ \mathrm{GeV}^6,\\
|Z_{\Xi_{bb}}|^2 &= (4.2 \pm 0.7) \times 10^{-2}\ \mathrm{GeV}^6.
\end{aligned}
\tag{42}
$$

In Fig. 10 we present the sum rules results for the ratio of baryonic constants $|Z_{\Omega_{bc}}|^2/|Z_{\Xi_{bc}}|^2$. We have also found

$$
\begin{aligned}
|Z_{\Omega_{bc}}|^2/|Z_{\Xi_{bc}}|^2 &= |Z_{\Omega_{cc}}|^2/|Z_{\Xi_{cc}}|^2\\
&= |Z_{\Omega_{bb}}|^2/|Z_{\Xi_{bb}}|^2\\
&= 1.3 \pm 0.2.
\end{aligned}
$$

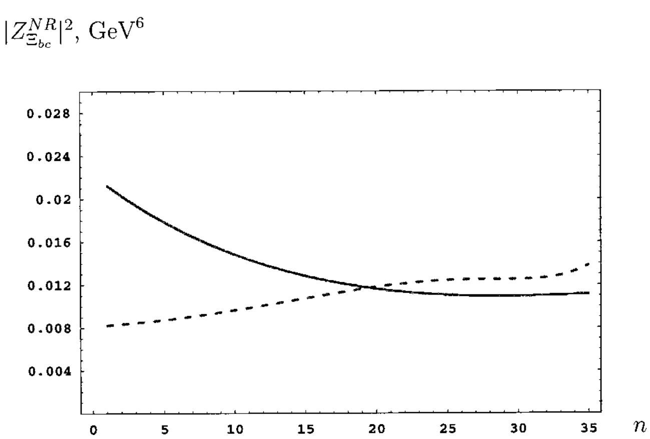

FIG. 9. The couplings $|Z^{(1,2)}_{\Xi_{bc}}|^2$ calculated in NRQCD sum rules for the form factors F_1 and F_2 (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.

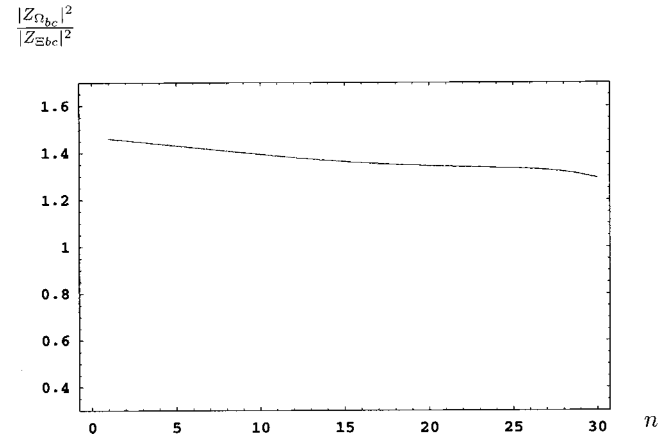

FIG. 10. The ratio $|Z_{\Omega_{bc}}|^2/|Z_{\Xi_{bc}}|^2$ calculated in NRQCD sum rules in the scheme of moments for the spectral densities at $\langle \bar{s}s\rangle/\langle \bar{q}q\rangle = 0.8$.

The uncertainty of this result, as mentioned above, is mainly connected with the little known ratio of $\langle \bar{s}s\rangle/\langle \bar{q}q\rangle = 0.8 \pm 0.2$.

For the sake of comparison, we derive the relation between the baryon coupling and the wave function of the doubly heavy baryon evaluated in the framework of potential model, where we have used the approximation of quark-diquark factorization. So, we find

$$
|Z^{\mathrm{PM}}| = 2\sqrt{3}|\Psi_d(0)\Psi_{l,s}(0)|, \tag{43}
$$

where $\Psi_d(0)$ and $\Psi_{l,s}(0)$ denote the wave functions at the origin for the doubly heavy diquark and light (strange) quark-diquark systems, respectively. In the approximation used, the values of $\Psi(0)$ were calculated in Ref. [19] in the potential by Buchmüller-Tye [21], so that

$$
\begin{aligned}
\sqrt{4\pi}|\Psi_l(0)| &= 0.53\ \mathrm{GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_s(0)| &= 0.64\ \mathrm{GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_{cc}(0)| &= 0.53\ \mathrm{GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_{bc}(0)| &= 0.73\ \mathrm{GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_{bb}(0)| &= 1.35\ \mathrm{GeV}^{3/2}.
\end{aligned}
$$

In the static limit of potential models, these parameters result in the estimates

$$
\begin{aligned}
|Z^{\mathrm{PM}}_{\Omega_{cc}}|^2 &= 8.8 \times 10^{-3}\ \mathrm{GeV}^6, \quad |Z^{\mathrm{PM}}_{\Xi_{cc}}|^2 = 6.0 \times 10^{-3}\ \mathrm{GeV}^6,\\
|Z^{\mathrm{PM}}_{\Omega_{bc}}|^2 &= 1.6 \times 10^{-2}\ \mathrm{GeV}^6, \quad |Z^{\mathrm{PM}}_{\Xi_{bc}}|^2 = 1.1 \times 10^{-2}\ \mathrm{GeV}^6,\\
|Z^{\mathrm{PM}}_{\Omega_{bb}}|^2 &= 5.6 \times 10^{-2}\ \mathrm{GeV}^6, \quad |Z^{\mathrm{PM}}_{\Xi_{bb}}|^2 = 3.9 \times 10^{-2}\ \mathrm{GeV}^6.
\end{aligned}
\tag{44}
$$

The estimates in the potential model (44) are close to the values obtained in the sum rules of NRQCD (42). We also see that the SU(3)-flavor splitting for the baryonic constants $|Z_\Omega|^2/|Z_\Xi|^2$ is determined by the ratio $|\Psi_s(0)|^2/|\Psi_l(0)|^2$

$=1.45$, which is in agreement with the sum-rules result. The values obtained in the NRQCD sum rules have to be multiplied by the Wilson coefficients coming from the expansion of full QCD operators in terms of NRQCD fields, as they have been estimated by use of corresponding anomalous dimensions. This procedure results in the final estimates

$$\begin{aligned}
|Z_{\Omega_{cc}}|^2 &= (38\pm 6)\times 10^{-3}\ \mathrm{GeV}^6,\\
|Z_{\Xi_{cc}}|^2 &= (27\pm 4)\times 10^{-3}\ \mathrm{GeV}^6,\\
|Z_{\Omega_{bc}}|^2 &= (36\pm 5)\times 10^{-3}\ \mathrm{GeV}^6,\\
|Z_{\Xi_{bc}}|^2 &= (27\pm 4)\times 10^{-3}\ \mathrm{GeV}^6,\\
|Z_{\Omega_{bb}}|^2 &= (10\pm 2)\times 10^{-2}\ \mathrm{GeV}^6,\\
|Z_{\Xi_{bb}}|^2 &= (70\pm 9)\times 10^{-3}\ \mathrm{GeV}^6.
\end{aligned} \tag{45}$$

IV. CONCLUSION

In this paper the NRQCD sum rules applied to the doubly heavy baryons have been considered. The nonrelativistic approximation for the heavy quark fields allows us to fix the structure of baryonic currents (the light quark-doubly heavy diquark) and to take into account the Coulomb-like interactions inside the doubly heavy diquark. The presence of both the nonzero mass of light quark and the contribution of nonperturbative terms of the quark, gluon, mixed condensates, and the product of condensates, destroys the factorization of the correlators. This fact provides the convergency of sum rules for each correlator and allows us to obtain reliable results for the masses and baryonic constants, which agree with the estimates in the framework of potential models. We also have calculated the mass splitting of Ω and Ξ doubly heavy baryons and the ratio of baryonic constants $|Z_\Omega|^2/|Z_\Xi|^2$.

ACKNOWLEDGMENTS

The authors are grateful to Professor A. K. Likhoded for stimulating discussions. This work is in part supported by the Russian Foundation for Basic Research, Grant Nos. 99-02-16558 and 00-15-96645.

APPENDIX

Here the derivation of expansion (6) is briefly presented. The calculations are done in the technique of fixed point gauge [22], so we write down the expansion of the quark field:

$$q(x)=q(0)+x^\alpha D_\alpha q(0)+\frac{1}{2}x^\alpha x^\beta D_\alpha D_\beta q(0)+\cdots,$$

and in the evaluation of $\langle 0|Tq_i^a(x)\bar{q}_j^b(0)|0\rangle$, where i and j are the spinor indices, and a, b are the color indices, we have to know how to get the vacuum average of type $\langle 0|D_\alpha\ldots D_\omega q(0)\bar{q}(0)|0\rangle$. The main formulas are the following:

The definitions of condensates

$$\langle q_i^a(0)\bar{q}_j^b(0)\rangle_0=-\frac{1}{12}\delta^{ab}\delta_{ij}\langle\bar{q}q\rangle,$$

$$\langle G^a_{\alpha\beta}G^{\alpha'}_{\alpha'\beta'}\rangle=\frac{\delta^{aa'}}{96}(g_{\alpha\alpha'}g_{\beta\beta'}-g_{\alpha\beta'}g_{\alpha'\beta})\langle G^2\rangle,$$

$$\langle\bar{q}igG^a_{\alpha\beta}t^a\sigma_{\alpha\beta}q\rangle_0=m_0^2\langle\bar{q}q\rangle,$$

the commutator of covariant derivatives

$$[D_\alpha,D_\beta]=-igG^a_{\alpha\beta}t^a,$$

and the equation of motion for the spinor field

$$\not{D}q=-im_q q.$$

From the last two equations we derive the so-called quadratic Dirac equation

$$D^2q=-m_q^2q+\frac{\sigma_{\alpha\beta}}{2}igG^a_{\alpha\beta}t^aq.$$

Now it is an easy challenge to obtain the first term in expansion (6).

Since the tensor $x_\alpha\ldots x_\omega$ is the symmetric one, we may perform the symmetrization

$$D_\alpha\ldots D_\omega\to\{D_\alpha,\ldots,D_\omega\}_+,$$

to find the nth term of expansion for $\langle\bar{q}(x)q(0)\rangle$, which equals

$$\frac{1}{n!}x_\alpha\ldots x_\omega\langle\bar{q}(0)D_\alpha\ldots D_\omega q(0)\rangle$$
$$=\frac{1}{n!}x_\alpha\ldots x_\omega\langle\bar{q}(0)\{D_\alpha,\ldots,D_\omega\}_+q(0)\rangle.$$

Note, the tensor $\langle\bar{q}(0)\{D_\alpha\ldots D_\omega\}_+q(0)\rangle$ is also a symmetric one.

The second term of expansion is derived from

$$\langle\{D_\alpha,D_\beta\}_+q^i_\rho(0)\bar{q}^j_\eta(0)\rangle=-2!\mathcal{P}_2g_{\alpha\beta}\delta^{ij}\delta_{\rho\eta}\langle\bar{q}q\rangle,$$

and the coefficient $\mathcal{P}_2$ is determined by contracting the indices α, β and using the quadratic Dirac equation

$$\mathcal{P}_2=(m_0^2-2m_q^2)/192.$$

The third term can be derived from the following structure:

$$\langle\{D_\alpha,D_\beta,D_\delta\}_+q^i_\rho(0)\bar{q}_\eta(0)\rangle$$
$$=-3!\mathcal{P}_3\delta^{ij}[(\gamma_\alpha)_{\rho\eta}g_{\beta\delta}+(\gamma_\beta)_{\rho\eta}g_{\alpha\delta}$$
$$+(\gamma_\delta)_{\rho\eta}g_{\alpha\beta}]\langle\bar{q}q\rangle.$$

Then, contracting α and β and using the equation of motion, the quadratic Dirac equation, and the commutation relation, we obtain

$$\mathcal{P}_3 = -i m_q (3m_0^2/4 - m_q^2)/576.$$

This includes the evaluation of vacuum averages

$$\langle D^2 D_\alpha q(0)\overline{q}(0)\rangle, \quad \langle D_\beta D_\alpha D_\beta q(0)\overline{q}(0)\rangle,$$

and

$$\langle D_\alpha D^2 q(0)\overline{q}(0)\rangle.$$

Considering the structure

$$\langle\{D_\alpha, D_\beta, D_\delta, D_\xi\}+q_\rho^i(0)\overline{q}_\eta(0)\rangle$$
$$= -4!\mathcal{P}_4 \delta^{ij}\delta_{\rho\eta}(g_{\alpha\beta}g_{\delta\xi}+g_{\alpha\delta}g_{\beta\xi}+g_{\alpha\xi}g_{\delta\beta})\langle\overline{q}q\rangle$$

contracted over any pair of indices, we derive

$$\mathcal{P}_4 = [\pi^2\langle\alpha_3/\pi G^2\rangle + 3/2 m_q^2(m_q^2 - m_0^2)]/3456.$$

Here we evaluated the following types of vacuum expectations:

$$\langle D^2 D^2 q(0)\overline{q}(0)\rangle, \quad \langle D_\alpha D_\beta D_\alpha D_\beta q(0)\overline{q}(0)\rangle,$$
$$\langle D_\alpha D^2 D_\alpha q(0)\overline{q}(0)\rangle.$$

Then the OPE for the quark condensate can be expressed in terms of $\mathcal{P}_i$ by

$$\langle q_\rho^i(x)\overline{q}_\eta^j(0)\rangle = -\delta^{ij}\langle\overline{q}q\rangle(\mathcal{P}_0\delta_{\rho\eta}+\mathcal{P}_1 x_\alpha\gamma^\alpha_{\rho\eta}+\mathcal{P}_2\delta_{\rho\eta}x^2 + \mathcal{P}_3 x_\alpha\gamma^\alpha_{\rho\eta}x^2+\mathcal{P}_4\delta_{\rho\eta}x^4),$$

with $\mathcal{P}_0 = 1/12$, and $\mathcal{P}_1 = -i m_q/48$.

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